Conical singularities regularized in warped six-dimensional flux compactification and their induced brane cosmology

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Abstract.

We discuss the regularization of codimension-2 singularities in warped six-dimensional Einstein-Maxwell axisymmetric models by replacing them by codimension-1 branes of a ring form, situated around the axis of symmetry. Further we consider the case of one capped regularized conical brane of codimension one and study the cosmological evolution which is induced on it as it moves in between the known *static* bulk and cap solutions. We present the resulting brane Friedmann equation which gives a dominant five-dimensional ρ^2 energy density term at high energies and a term linear to the energy density at low energies with, however, negative coefficient in the small four-brane radius limit (*i.e.,* with negative effective Newton's constant).

1. Introduction

Cosmology in theories with branes embedded in extra dimensions has been the subject of intense investigation during the last years. The most detailed analysis has been done for braneworld models in five-dimensional space [1]. The effect of the extra dimension can modify the cosmological evolution, depending on the model, both at early and late times. The cosmology of this and other related models with one transverse to the brane extra dimension (codimension-1 brane models) is well understood [2, 3, 4, 5, 6, 7, 8] (for a review see [9]).

Braneworld models can also be extended in higher than five-dimensions. We can consider a $(n+1)$ brane embedded in $(n+2)$ spacetime (codimension-1 models) or a $(3+1)$ brane embedded in $(n+1)$ spacetime (codimension-$n-3$ models). Six or higher-dimensional braneworld models of codimension-1 are considered as generalizations of the Randall-Sundrum model [10].

Six-dimensional brane world models are rather interesting to be studied for a number of reasons. Firstly the large (sub-millimeter) extra dimensions proposal for the resolution of

the electroweak hierarchy problem was provided [11] in the framework of brane theories with two extra transverse dimensions. Therefore the study of extra dimensional theories, and string theory in particular, become relevant to low energy phenomenology (colliders as well as astrophysical and cosmological observations) and is testable in the very near future. The different possibilities of realizing a brane world model in six dimensions must therefore be studied to allow for a comparison with experiment.

Another attractive motivation has been the proposal to ameliorate the cosmological constant problem, using codimension-2 branes (for a recent review on the subject see [12]). These branes have the interesting property that their vacuum energy instead of curving their world-volume, just introduces a deficit angle in the local geometry [13]. Models with this property which exhibit no fine-tuning between the brane and bulk quantities have been known as self-tuning (for early attempts to find similar models in five dimensions see [14]). Such self-tuning models with flux compactification [15, 16] have been extensively looked, but the flux quantization condition always introduces a fine tuning [17], unless one allows for singularities more severe than conical [18]. Alternative sigma-model compactifications have been shown to satisfy the self-tuning requirements [19]. However, the successful resolution of the cosmological constant problem would also require that there are no fine-tuning between bulk parameters themselves. No such self-tuning model has been found yet with all these properties.

A further motivation in studying such models with codimension-2 branes is that gravity on them is purely understood. The introduction of matter (*i.e.,* anything different from vacuum energy) on them, immediately introduces malicious non-conical singularities [20]. A way out of this problem is to complicate the gravity dynamics by adding a Gauss-Bonnet term in the bulk or an induced curvature term on the brane, in which case the singularity structure of the theory is altered and non-trivial matter is allowed [21]. However, the components of the energy-momentum tensor of the brane and the bulk are tuned artificially and the brane matter is rather restricted [22]. Alternatively, one can regularize the codimension-2 branes by introducing some thickness and then consider matter on them [23]. For example, one can mimic the brane by a six-dimensional vortex (as *e.g.,* in [24]), a procedure which becomes a rather difficult task if matter is added on it.

Even thought we do not fully understand black hole solutions in codimension-2 braneworlds they have been extensively discussed in [25, 26, 27, 33]. Lately a six-dimensional black hole localized on a 3-brane of codimension-2 [26, 27] was proposed. However, it is not clear how to realize these solutions in the thin brane limit where high curvature terms are needed to accommodate matter on the brane. The localization of a black hole on the brane and its extension to the bulk is a difficult task. In codimension-1 braneworlds the first attempt was to consider the Schwarzschild metric and study its black string extension into the bulk [28]. which is unfortunately unstable to classical linear perturbations [29]. Since then, several authors have attempted to find the full metric using numerical techniques [30]. Analytically, the brane metric equations of motion were considered with the only bulk input coming from the projection of the Weyl tensor [31] onto the brane. Since this system is not closed because it contains an unknown bulk dependent term, assumptions have to be made either in the form of the metric or on the Weyl term [32]. In codimension-2 braneworlds, recently black holes on a thin conical brane and their extension into a five and six-dimensional bulk with a Gauss-Bonnet term were found [33].

Moreover we are still lacking an understanding of time dependent cosmological solutions in codimension-2 braneworlds. To have a cosmological evolution we need regularized branes, the brane world-volume should be expanding and in general the bulk space should also evolve

in time. This is a formidable task, so we follow an alternative approach [34, 35]. [1]

First for our setup we consider another way of regularization which was recently proposed and consists merely of the reduction of codimensionality of the brane. In this approach, the bulk around the codimension-2 brane is cut close to the conical tip and it is replaced by a codimension-1 brane which is capped by appropriate bulk sections [37] (see [38] for a similar regularization of cosmic strings in flat spacetime). This regularization has been applied to flux compactification systems in six dimensions for unwarped "rugby-ball"-like solutions in [37]. The case of warped solutions with conical branes (with or without supersymmetry) [39] and even more general warped solutions allowing non-conical branes [40] have been studied. More precisely we consider the case of one capped regularized conical brane of codimension one and give the *static* bulk and cap solutions.

Then we study the motion of the regularized codimension-1 brane in the space between the bulk and the brane-cap which remains static (see *e.g.,* [41]). In this way, a cosmological evolution will be induced on the brane in a similar way as in the mirage cosmology [42], but with the inclusion of the back-reaction of the brane energy density (*i.e.,* the brane is not considered merely a probe one). Since in the mirage cosmology, the four-dimensional scale factor descends from the warp factor in the four-dimensional part of the bulk metric, we will discuss the regularized brane in the case of warped bulk [39], rather than unwarped bulk [37]. It is worth noting that the above procedure provided in five dimensions the most general isotropic brane cosmological solutions [3].

We find the Friedmann equation on the brane by solving the Israel junction conditions, which play the rôle of the equations of motion of the codimension-1 brane. We find that at early times cosmology is dominated by an energy density term proportional to ρ^2, like in the Rundall-Sundrum model in five dimensions. However, at late times where the brane moves close to its equilibrium point, which in turn is close to the would-be conical singularity, the coefficient of the linear to the energy density term is negative (*i.e.,* we obtain negative effective Newton's constant). Thus, we cannot recover the standard four dimensional cosmology at late times. This seems to be the consequence of considering the bulk sections static. It is possible, that this behaviour is due to a ghost mode appearing among the perturbations of the system, after imposing the staticity of the bulk sections. Furthermore, the above result points out that there is a difference between the six-dimensional brane cosmology in comparison to the five-dimensional one. The study of brane cosmology in Einstein gravity in five dimensions, can be made either in a gauge where the bulk is time-dependent and the brane lies at a fixed position, or in a gauge where the bulk is static and the brane movement into the bulk induces a cosmological evolution on it [43]. This, however, does not seem to hold in six-dimensions anymore.

The talk is organized as follows. In Sec. 2 we present the static regularized brane solution in a bulk of general warping. In Sec. 3 we derive the equations of motion of the codimension-1 brane and in Sec. 4 we study the induced cosmological evolution on the moving brane. Finally in Sec. 5 we draw our conclusions.

2. Regularized static brane solutions

In this section we will briefly give the necessary results of the static solution which we will need in the following for the brane motion. The bulk theory that we use is a six-dimensional

[1] Recently, some works have studied the cosmological evolution of codimension-1 or 2 branes in six dimensional models [36]

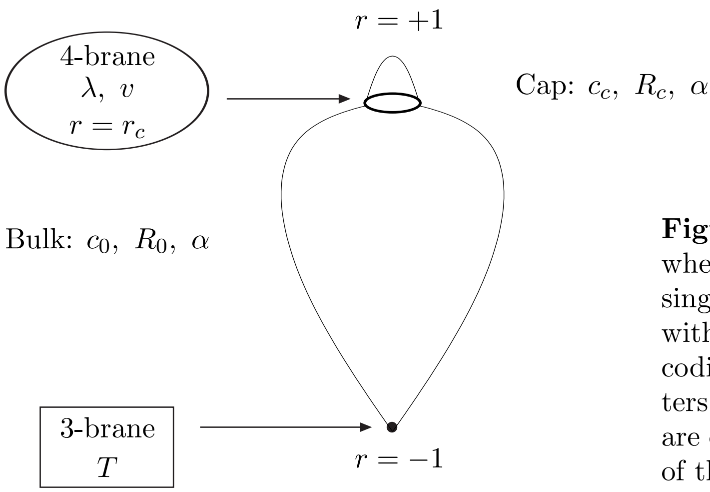

Figure 1. The internal space where the upper codimension-2 singularity has been regularized with the introduction of a ring-like codimension-1 brane. The parameters of the action and the solution are denoted in the appropriate part of the internal space.

Einstein-Maxwell system which in the presence of a positive cosmological constant and a gauge flux, spontaneously compactifies the internal space [44]. The known axisymmetric solutions have in general two codimension-2 singularities at the poles of a deformed sphere [45]. We study the case where only one (*e.g.,* the upper) codimension-2 brane is regularized by the introduction of a ring-like brane at $r = r_c$ with an appropriate cap. The dynamics of the system is given by the following action

$$S = \int d^6x\sqrt{-g}\left(\frac{M^4}{2}R - \Lambda_i - \frac{1}{4}\mathcal{F}^2\right) - \int d^5x\sqrt{-\gamma_+}\left(\lambda + \frac{v^2}{2}(\tilde{D}_{\hat{\mu}}\sigma)^2\right) - \int d^4x\sqrt{-\gamma_-}\,T\ , \quad (1)$$

where M is the six-dimensional fundamental Planck mass, Λ_i are the bulk ($i = 0$) and cap ($i = c$) cosmological constants, $\mathcal{F}_{MN}$ the gauge field strength, T the tension of the lower codimension-2 brane, λ the 4-brane tension, σ the 4-brane Goldstone scalar field necessary for the regularization and v the vev of the Higgs field from which the Goldstone field originates. For the coupling between the Goldstone field and the bulk gauge field we use the notation $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - e\ a_{\hat{\mu}}$, with $a_{\hat{\mu}} = \mathcal{A}_M\partial_{\hat{\mu}}X^M$ the pullback of the gauge field on the ring-like brane and e its coupling to the scalar field. In the above action we omitted the Gibbons-Hawking term. The configuration is shown in more detail in Fig.1.

The solution for the bulk and cap regions depends on a parameter α which is a measure of the warping of the space (for $\alpha = 1$ we obtain the unwarped case) and is given by [39]

$$ds_6^2 = z^2\eta_{\mu\nu}dx^\mu dx^\nu + R_i^2\left[\frac{dr^2}{f} + c_i^2 f\ d\varphi^2\right]\ , \quad (2)$$

$$\mathcal{F}_{r\varphi} = -c_i R_i M^2 S \cdot \frac{1}{z^4}\ , \quad (3)$$

with $R_i^2 = M^4/(2\Lambda_i)$ and the following bulk functions

$$z(r) = \frac{1}{2}[(1-\alpha)r + (1+\alpha)] \quad (4)$$

$$f(r) = \frac{1}{5(1-\alpha)^2}\left[-z^2 + \frac{1-\alpha^8}{1-\alpha^3}\cdot\frac{1}{z^3} - \alpha^3\frac{1-\alpha^5}{1-\alpha^3}\cdot\frac{1}{z^6}\right]\ , \quad (5)$$

with $S(\alpha) = \sqrt{\frac{3}{5}\alpha^3 \frac{1-\alpha^5}{1-\alpha^3}}$. The range of the internal space coordinates is $-1 \leq r \leq 1$ and $0 \leq \varphi < 2\pi$. Taking into account that in the limit $r \to \pm 1$, it is $f \to 2(1 \mp r) X_\pm$ with the constants $X_\pm$ given by

$$X_+ = \frac{5 + 3\alpha^8 - 8\alpha^3}{20(1-\alpha)(1-\alpha^3)} \quad , \quad X_- = \frac{3 + 5\alpha^8 - 8\alpha^5}{20\alpha^4(1-\alpha)(1-\alpha^3)} \, , \tag{6}$$

the cap is smooth at $r = +1$ as long as is $c_c = 1/X_+$. Furthermore, the metric is continuous if $c_0 R_0 = c_c R_c$, which gives $R_c = \beta_+ R_0$ with $\beta_+ = X_+ c_0$ [2]. The conical singularity at $r = -1$ is supported by a codimension-2 brane with tension

$$T = 2\pi M^4 (1 - c_0 X_-) \tag{7}$$

while the parameters of the 4-brane λ, v are fixed by the radii R_0, R_c and the brane position r_c [39]. Finally, the gauge field is quantized as

$$2 c_0 R_0 M^2 e \, Y = N \quad , \quad N \in \mathbb{Z} \, , \tag{8}$$

with $Y = \frac{(1-\alpha^3)}{3\alpha^3(1-\alpha)} \, S$ and the brane scalar field has solution $\sigma = n\varphi$ with $n \in \mathbb{Z}$. The two quantum numbers n, N are related through the junction conditions as

$$n = \frac{N}{2} \frac{2}{(1-\alpha^3)} \left[\frac{5(1-\alpha^8)}{8(1-\alpha^5)} - \alpha^3 \right] \, . \tag{9}$$

Since the quantities n, N are integers, the above relation imposes a restriction to the values of the admissible warpings α, which implies that static solutions are consistent only for discrete values of the warping α.

3. The cosmological dynamics of the 4-brane

To study the cosmological evolution on the brane we include matter on it which in general makes both the brane and the bulk to evolve in time. Unfortunately we do not know any time dependent solution for such setup. Therefore we will approach the problem in a simplest way, by making the approximation that the bulk remains *static* and that the brane matter merely makes the brane to move between the static bulk and the static cup, away from its equilibrium point $r = r_c$, thus obtaining cosmology on the brane, as in the mirage cosmology approach [42]. This is to be regarded as a first step towards understanding the generic brane cosmological evolution. In our case the brane is not merely a probe brane and we will use the junction conditions to derive the induced cosmology, thus taking into account the back-reaction of the brane energy density.

Firstly, to embed the brane in the static bulk, let us take the brane coordinates be $\sigma^{\hat{\mu}} = (\sigma, x^i, \varphi)$. [The brane-time coordinate σ is not to be confused with the Goldstone field σ which will not appear in our subsequent analysis.] Then the brane embedding X^M in the bulk is taken for both sections to be

$$X^i = x^i \quad , \quad X^r = \mathcal{R}(\sigma) \quad \text{and} \quad X^\varphi = \varphi \, , \tag{10}$$

[2] In this brief presentation of the background, we have taken $\xi = 1$ in comparison with [39]. The physical quantities, however, are ξ-independent and depend only on β_+.

while for the time coordinate embedding we choose for the outer bulk section

$$X^0_{(out)} = \sigma \ , \tag{11}$$

and for the inner cap section

$$X^0_{(in)} = T(\sigma) \ . \tag{12}$$

The continuity of the induced metric $\gamma_{\hat{\mu}\hat{\nu}} = g_{MN}\partial_{\hat{\mu}}X^M\partial_{\hat{\nu}}X^N$, apart from the relation $c_0R_0 = c_cR_c$ as in the static case, gives a relation of the time coordinate T in the upper cap region with the brane time coordinate σ (dots are with respect to σ)

$$\dot{T}^2\left(1-\beta_+^2\frac{\dot{\mathcal{R}}^2}{\dot{T}^2}\frac{R_0^2}{fz^2}\right) = \left(1-\dot{\mathcal{R}}^2\frac{R_0^2}{fz^2}\right) \ . \tag{13}$$

Then the induced metric $\gamma_{\hat{\mu}\hat{\nu}}$ on the brane reads

$$ds^2_{(5)} = -z^2\left(1-\dot{\mathcal{R}}^2\frac{R_0^2}{fz^2}\right)d\sigma^2 + z^2d\vec{x}^2 + c_0^2R_0^2fd\varphi^2 \ . \tag{14}$$

The continuity of the gauge field, on the other hand, is guaranteed by the fact that its only non-vanishing component is A_φ and X^φ is σ-independent.

Secondly, we introduce an energy momentum tensor of a perfect fluid on the brane $t_{\hat{\mu}}^{\hat{\nu}\ (br)} = -(2/\sqrt{-\gamma_+})\delta S_{br}/\delta\gamma_+{}^{\mu}{}_{\nu} = \mathrm{diag}(-\rho, P, P, P, \hat{P})$ (where S_{br} is the brane action). We also consider a possible coupling of the brane matter to the bulk gauge field (consistent with the cosmological symmetries) by $\delta S_{br}/\delta a^{\hat{\kappa}} = (l, L, L, L, \hat{L})$. Splitting the above quantities to one part responsible for the static solution and another expressing the presence of matter on the brane, we have

$$\begin{aligned}
\rho &= \lambda + \frac{v^2(n-e\mathcal{A}_\varphi^+)^2}{2c_0^2R_0^2f(r_c)} + \rho_m \equiv \rho_0 + \rho_m & (15)\\
P &= -\lambda - \frac{v^2(n-e\mathcal{A}_\varphi^+)^2}{2c_0^2R_0^2f(r_c)} + P_m \equiv -\rho_0 + P_m & (16)\\
\hat{P} &= -\lambda + \frac{v^2(n-e\mathcal{A}_\varphi^+)^2}{2c_0^2R_0^2f(r_c)} + \hat{P}_m & (17)\\
l &= l_m & (18)\\
L &= L_m & (19)\\
\hat{L} &= ev^2(n-e\mathcal{A}_\varphi^+) + \hat{L}_m & (20)
\end{aligned}$$

with all the other quantities vanishing and ρ_m, P_m, $\hat{P}_m$, l_m, L_m, $\hat{L}_m$ the matter contributions.

In the spirit of mirage cosmology approach [42] the brane matter merely makes the brane to move in the static bulk. As we have already stated this movement induces cosmology on the brane and warping of the bulk is necessary. Therefore it is convenient to rewrite the metric in the form

$$ds^2_{(5)} = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)d\varphi^2 \ , \tag{21}$$

with $a = z(\mathcal{R}(\tau))$ and $b = c_0 R_0 \sqrt{f(\mathcal{R}(\tau))}$. The brane proper time is given by

$$\dot{\tau}^2 = z^2 \left(1 - \dot{\mathcal{R}}^2 \frac{R_0^2}{f z^2}\right) \ . \tag{22}$$

From now on we will assume without loss of generality that $\dot{\tau} > 0$. It is evident from the above, that cosmological evolution from mere motion of the brane in the static bulk is possible only when there is warping in the bulk. In the unwarped version of this model [37], mirage cosmology is impossible and some bulk time-dependence is compulsory.

The Hubble parameters for the two scale factors are given by

$$H_a \equiv \frac{1}{a}\frac{da}{d\tau} = \frac{z'}{z^2} \frac{\dot{\mathcal{R}}}{\sqrt{1 - \dot{\mathcal{R}}^2 \frac{R_0^2}{f z^2}}} \ , \tag{23}$$

and

$$H_b \equiv \frac{1}{b}\frac{db}{d\tau} = \frac{f'}{2fz} \frac{\dot{\mathcal{R}}}{\sqrt{1 - \dot{\mathcal{R}}^2 \frac{R_0^2}{f z^2}}} \ . \tag{24}$$

Then the ratio of H_a and H_b gives a precise relation between them for our particular model. It is given by

$$H_b = \frac{z f'}{2 f z'} H_a \ , \tag{25}$$

and we notice that since in the model we study it is always $f' < 0$, in the neighborhood of $r = 1$, the two Hubble parameters have opposite sign. This means that if the four dimensional space expands, the internal space shrinks.

Apart from the continuity conditions we have to take into account the junction conditions for the derivatives of the metric and the gauge field, which read

$$\{\hat{K}_{\hat{\mu}\hat{\nu}}\} = -\frac{1}{M^4} t^{(br)}_{\hat{\mu}\hat{\nu}} \ , \tag{26}$$

$$\{n_M \mathcal{F}^M_{\ N} \partial_{\hat{\kappa}} X^N\} = -\frac{\delta S_{br}}{\delta a^{\hat{\kappa}}} \ . \tag{27}$$

We denote $\{H\} = H^{in} + H^{out}$ the sum of the quantity H from each side of each brane. The extrinsic curvatures are constructed using the normal to the brane n_M which points *inwards to the corresponding part of the bulk* each time (we use the conventions of [46]). [3].

So far we have 6 parameters: $\rho, P, \hat{P}, l, L, \hat{L}$ and with the use of the continuity relations of the induced metric, $c_0 R_0 = c_c R_c$ and (13), the (t) and (i) components of the junction conditions for the gauge field give for the coupling: $l = L = 0$, the (φ) component of the gauge field junction and the $(\varphi\varphi)$ component of the metric junction determine $\hat{P}$ and $\hat{L}$. Finally the two remaining metric junction conditions ($(\sigma\sigma)$ and (ij) components) give the Friedmann equation and the acceleration equation respectively.

The obtained Friedmann equation is

$$H_a^2 = \frac{1}{4M^8 \mathcal{C}^2 \mathcal{A}}\ \rho^2 + \frac{M^8 \mathcal{C}^2 (1-\beta_+^2)^2}{4 R_0^4 \beta_+^4 \mathcal{A}} \cdot \frac{1}{\rho^2} - \frac{1+\beta_+^2}{2\beta_+^2 R_0^2 \mathcal{A}} \tag{28}$$

[3] The left hand sides of the above equations are computed in detail in the Appendix A of [34]

where $\mathcal{A} = \frac{z^2}{fz'^2}$ and $\mathcal{C} = \sqrt{f}\left(3\frac{z'}{z} + \frac{f'}{2f}\right)$, both evaluated on the brane. It has unconventional dependence on the energy density, in particular the inverse square dependence is known to occur for motions in backgrounds of asymmetrical warping [47].

The equilibrium point r_c of the system is found if we set $H_a = 0$, which gives the brane energy density without matter

$$\rho_0 = -\frac{M^4 \mathcal{C}_c}{R_0 \beta_+}(1-\beta_+) \ , \tag{29}$$

where $\mathcal{C}_c$ is the value of $\mathcal{C}$ at r_c and is the same appearing in (15). The behaviour of the function $\mathcal{C}$ is that, as $r \to -1$ it limits to $\mathcal{C} \to \infty$ and monotonically decreases and limits to $\mathcal{C} \to -\infty$ as $r \to 1$. On the other hand $\mathcal{A}$, is always positive with $\mathcal{A} \to \infty$ as $r \to \pm 1$, and $\mathcal{O}(1)$ in the intermediate region. In the unwarped limit $\alpha \to 1$ the Friedmann equation becomes as expected trivial, *i.e.*, $H_a = 0$.

Before studying various limits of the above equation, let us define the effective four dimensional matter energy density $\rho_m^{(4)}$ by averaging over the azimuthal direction (we assume that ρ_m is independent of φ)

$$\rho_m^{(4)} = \int d\varphi \sqrt{g_{\varphi\varphi}}\rho_m = \frac{2\pi\beta_+}{X_+} R_0 \sqrt{f} \rho_m \ , \tag{30}$$

with similar definitions for the other 4-brane quantities.

Let us suppose now that initially $-1 \ll \mathcal{R}(\sigma) < r_c < 1$, with $1 - r_c \ll 1$. The goal is to find how $\mathcal{R}(\sigma)$ behaves. To recover a four-dimensional Friedmann equation at late times we can assume that the brane energy density is small in comparison with the static case energy density, *i.e.*, $\rho_m^{(4)} \ll \rho_0$, so we can expand (28) in powers of $\rho_m^{(4)}$ and obtain the following four dimensional form of the Friedmann equation

$$H_a^2 = \frac{8\pi}{3} G_{eff} \rho_m^{(4)} + \Delta(a) + \mathcal{O}(\rho_m^{(4)\ 2}) \ , \tag{31}$$

where the effective Newton's constant is

$$G_{eff} = \frac{3X_+ \mathcal{C}_c (1-\beta_+)}{32\pi^2 R_0^2 \beta_+^2 M^4 \mathcal{A}\, \mathcal{C}^2 \sqrt{f}} \left[\frac{\mathcal{C}^4}{\mathcal{C}_c^4}\left(\frac{1+\beta_+}{1-\beta_+}\right)^2 - 1\right] \ . \tag{32}$$

The quantity $\Delta(a)$ depends on the parameters of the bulk and plays the rôle of the the mirage matter induced on the brane from the bulk and it is given by

$$\Delta(a) = \frac{(1-\beta_+)^2}{4R_0^2 \beta_+^2 \mathcal{A}} \left[\frac{\mathcal{C}_c^2}{\mathcal{C}^2} + \frac{\mathcal{C}^2}{\mathcal{C}_c^2}\left(\frac{1+\beta_+}{1-\beta_+}\right)^2 - 2\frac{1+\beta_+^2}{(1-\beta_+)^2}\right] \ . \tag{33}$$

The behaviour of G_{eff} as a function of $\mathcal{R}(\sigma)$ is given in Fig. 2 and has the following important features: At the points where the geometry becomes conical ($r \to \pm 1$) the effective Newton's constant is negative and diverging. In between, there is a point r_d, with

$$z_d = \left(\frac{3(1-\alpha^8)}{8(1-\alpha^3)}\right)^{1/5} \ , \tag{34}$$

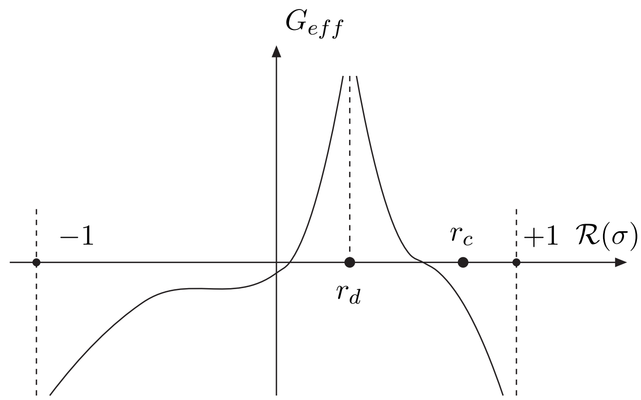

Figure 2. The generic form of the effective Newton's constant G_{eff} as a function of the brane position $\mathcal{R}(\sigma)$. As the brane approaches the equilibrium point r_c, we always have $G_{eff} < 0$. Additionally, G_{eff} diverges as $r \to \pm 1$ and at one point r_d in between.

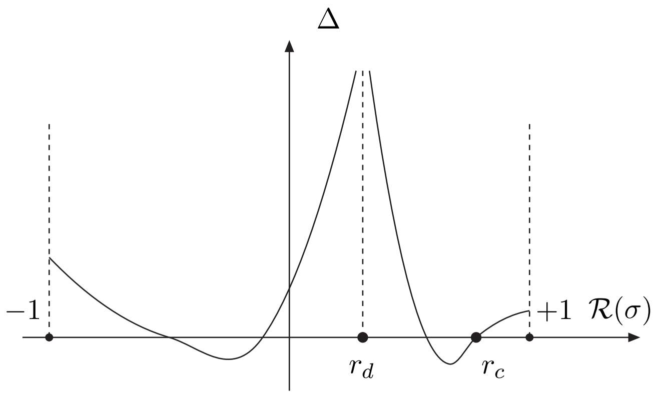

Figure 3. The generic form of the mirage matter contribution Δ as a function of the brane position $\mathcal{R}(\sigma)$. At the static equilibrium point r_c it vanishes and at r_d it diverges.

which is a root of $\mathcal{C}$ and G_{eff} diverges to $+\infty$. At this point the matter energy density is bound to vanish. It is important to note that even in the region where G_{eff} is positive, there is always a strong time variation of G_{eff}, which for

$$\frac{1}{G_{eff}} \frac{dG_{eff}}{d\tau} = H_a \delta \ , \tag{35}$$

has $\delta > \mathcal{O}(10)$, in contradiction with observations [48] which dictate that $\delta < 0.1$. But even more important is the fact that close to the static equilibrium point, where the cosmology is supposed to mimic best the one of a codimension-2 brane, we get *negative* Newton's constant

$$G_{eff}(\mathcal{R} = r_c) = \frac{3X_+}{8\pi^2 R_0^2 \beta_+ (1-\beta_+) M^4 \mathcal{A}\ \mathcal{C}_c \sqrt{f}} < 0 \ , \tag{36}$$

since we have that for any value of the parameters and for r_c in the neighborhood of $r = +1$, it is $\mathcal{C}_c < 0$.

Let us also look at the mirage matter contribution Δ, which is depicted in Fig. 3. As expected, it vanishes for the static equilibrium point r_c, it is finite at the boundaries $r = \pm 1$ and diverges at the root r_d of $\mathcal{C}$. Again there is no region in the brane position interval where the contribution of Δ is constant enough to resemble a cosmological constant contribution to the four dimensional Friedmann equation.

This result is not altered by supersymmetrizing the model [16]. The bulk and cap solutions are different from the non-supersymmetric case due to the presence of the dilaton. However,

the only difference in the Friedmann is a redefinition of the quantities $\mathcal{A}$ and $\mathcal{C}$. In the supersymmetric case, these quantities read

$$\mathcal{A}_{susy} = 4\frac{z^2}{f z'^2} \quad \text{and} \quad \mathcal{C}_{susy} = \sqrt{f}\left(\frac{3z'}{2z} + \frac{f'}{2f}\right) . \tag{37}$$

It is easy to see again that the effective Newton's constant is negative for the motion near the would-be conical tip and that even away from that point, it is very strongly varying.

On the opposite limit, that the matter energy density is much larger than the static case energy density, *i.e.*, $\rho_m^{(4)} \gg \rho_0$, we get the expected asymptotics

$$H_a^2 = \frac{1}{4M^8\mathcal{C}^2\mathcal{A}}\ \rho_m^2 , \tag{38}$$

which is a five-dimensional Friedmann law (with time-varying five-dimensional Newton's constant) at early times.

Taking under consideration the difficulties of the model to give a Fiedmann equation with the correct sign of G_{eff}, we will not proceed with the presentation of the analysis of the acceleration equation. With this equation, one finds even more difficulties towards obtaining a realistic four-dimensional evolution. For example, in the low energy limit, one gets a coefficient of the linear energy density term, which is not related to the G_{eff}, that we obtained from the Friedmann equation, in the way it does in standard four-dimensional General Relativity.

This pathological features of the low energy density limit, where the expansion does not appear to have an effective four-dimensional limit, can find a potential explanation when looking at the energy continuity equation on the brane. Taking the covariant divergence of the Israel junction condition (26), together with the Codazzi equation $\nabla^{(4)}_{\hat\mu}\hat{K}^{\hat\mu}_{\hat\nu} = G_{K\Lambda}n^\Lambda h^K_N\partial_{\hat\nu}X^N$ and the bulk Einstein equation, we arrive at the following simple equation [46]

$$\nabla^{(4)}_{\hat\mu} t^{\hat\mu\ (br)}_{\hat\nu} = -\{T^{(B)}_{K\Lambda}h^K_N n^\Lambda\partial_{\hat\nu}X^N\} . \tag{39}$$

Because of the jump of the bulk energy momentum tensor across the 4-brane, the energy-momentum tensor on the 4-brane is not conserved. This is a usual feature of moving brane cosmologies in asymmetrically warped backgrounds [49]. In more detail, the form of the above equation for the particular model is given by

$$\frac{d\rho}{d\tau} + 3(\rho + P)H_a + (\rho + \hat{P})\frac{zf'}{2fz'}H_a = -\frac{S^2\rho H_a}{z'z^7\mathcal{C}\sqrt{f}} , \tag{40}$$

where in the right hand side we have used the Friedmann equation (28). In a straightforward but lengthy calculation, one can see that the latter equation can be derived from the $(\sigma\sigma)$, (ij) and $(\varphi\varphi)$ components of the junction conditions of (26). The problems in the four-dimensional limit, that we faced previously, can be traced to large energy dissipation off the brane as well as a large work done during the contraction of the ring-brane.

4. Conclusions

In the present presentation we made a first step towards the study of the cosmology of a codimension-2 brane. We regularized the codimension-2 singularities by the method of

lowering its codimensionality. We cut the space close to the conical tip and replaced it by a ring brane with an appropriate cap. Unfortunately we do not know any time dependent solution for such setup. Therefore we will approach the problem in a simplest way, we assumed that the bulk and the cap remain static as the brane moves between them. The motion of the brane then induces a cosmological evolution for the matter on the brane. The junction conditions provide the Friedmann and acceleration equations on the brane.

From the Friedmann equation we can see that we cannot recover standard cosmological evolution of the brane at low energies. The effective Newton's constant is negative in the interesting limit that the brane approaches its equilibrium point, close to the would-be conical singularity. In other words, we obtain antigravity in this limit. Even away from this point, *i.e.,* when the brane moves away from its equilibrium point, the Newton's constant varies significantly in contradiction with standard cosmology. At one position of the internal space, the Newton's constant even diverges and forces the matter energy density to vanish. Taking all the above into account, we did not present the further analysis of the system by considering the acceleration equation.

The reason for this unconventional cosmological evolution, is the use of the specific restricted ansatz for the solution of the system's equation of motion. The staticity of the bulk was proved to be an oversimplification. We imagine that this restriction on the system may result to the appearance of some scalar mode in the perturbative analysis of [50], which for a certain region of the brane motion (close to the pole of the internal manifold) is ghost-like. This mode may then be responsible for the negative effective Newton's constant. The unrestricted perturbative analysis in [50] resulted in a linearized four-dimensional Einstein equation for distances larger than the compactification scale. The same happened in [37] in the unwarped model perturbation analysis[4] (see [51] for a related analysis with a brane induced gravity term), even though it would not have mirage evolution, as we studied it here, because of the absence of a warp factor. The gradient expansion technique was recently used in similar six dimensional models [52] to derive a low energy effective theory on the regularized brane and show that standard four-dimensional Einstein gravity is recovered at low energies. Consequently, this approach could be used in our case to derive the low energy effective theory [53]. The appearance of the standard four-dimensional linearized dynamics, shows that they are realized when the bulk is necessarily time-dependent.

Clearly, the next step should be the study of the system in a setup where the bulk is also time-dependent. In that respect, brane cosmology in six dimensions seems to be different for the one in five dimensions. In Einstein gravity in five dimensions, one can always work in a gauge where the bulk is static and the brane acquires a cosmological evolution by moving into the bulk. However, in six dimensions there are more degrees of freedom which make this gauge choice not general. In the present paper, we have frozen these degrees of freedom hoping to find consistent cosmological solutions, but it turned out that this does not give a viable cosmology. There are several known time-dependent backgrounds which can be used to look for realistic brane cosmologies [54]. We plan to address this issue in the near future.

Acknowledgments
This talk is based on the works done in collaboration with E. Papantonopoulos and A. Papazoglou. We thank the organizers of the 13th NEB Conference, held in Thessaloniki,

[4] The question of stability of the regularized models was not discussed in [37, 50] and it could be that these compactifications have modes with negative mass squared.

for their kind hospitality and perfect organization. This work was supported by the Hellenic Army Academy and the NTUA research program PEVE07.

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